Beyond the present understanding of spontaneous organization in fluids and plasmas: a field theoretical approach

Summary of the Final Report for the Project ERC – Like 4/2012 (Florin Spineanu)

The dynamics of the quasi-ideal fluids in two-dimensions show the tendency to self-organization. The flow which is initialized in an irregular pattern evolves in time to a highly ordered stationary structure. The characterization of the states is made on the basis of the usual functions: stream function, velocity and vorticity. To identify the asymptotic coherent states of flow, one notes the indefinite number of irreducible correlations (cumulants, specific to stationary structures) and, in addition, verifies that the stream function is the solution of the sinh-Poisson equation. Regarding the fundamental process that leads to self-organization, one can invoke the classical results of the theory of turbulence: inverse spectral cascade of energy and direct spectral cascade of the enstrophy.

But the tendency to order has also different stages than those where the language of turbulence is adequate. The late phases, preceding the stationary coherent states, consist of a small number of mesoscopic vortices that are slowly moving in plane. Their encounter and mergings generate the final order. This phase escapes the description in terms of turbulence. It is usually studied by numerical simulation and experiments.

In the study of the spontaneous evolution of the ideal 2D Euler fluid toward highly coherent flow pattern, the theory has adopted the turbulence point of view. The idea that lies behind the process has been treated in an indirect way: the fluid vorticity has been discretized into a set of point-like vortices and this system has been treated as a gas. The statistical approach has met certain difficulties: the phase space is finite, there is no thermodynamic limit, there is no ergodicity, the temperature is negative, the entropy is invoked in a counter-intuitive way, etc. but the statistical approach finds the answer: it derives the equation that is obeyed by the stream function in the asymptotic states: sinh-Poisson. We note that just at this moment it would have become evident that the exact integrability and the exceptional property of an algebraic-geometrical structure, the self-duality, are involved in the fluid's relaxation. A different language was necessary.

We have proposed for the Euler fluid an approach based on a classical field theory. We have constructed the Lagrangian such as to reflect: the vortical nature of the elements (mixed spinor from the algebra sl(2,C)); motion of the ExB –type (the term Chern-Simons); "long-range" interaction (adopting a particular matter-field self-interaction). We have derived the equations of motion, the currents and we have identified the extremum of the action, so offering a purely analytical derivation of the equation *sinh*-Poisson. The self-duality has emerged naturally, and this fundamental property that allows solitons, instantons, vortices, topological structures, has been associated with the ideal Euler fluid in 2D.

The field theory is formulated in terms of two fields: matter and gauge. These are actually different representations of a single physical object: the vorticity. Why is-it necessary to extend over two different representations? The reason follows the persistent endeavor of the fluid theory, i.e. the unfolding of the fundamental (Riemann) nonlinearity: the advection of the vorticity by its own velocity field. What the field theory does is the analogue of the Hopf-Cole transformation which unfolds the Riemann nonlinearity of the Burgers fluid in 1D.

We studied the states that precede the stationarity. There are two directions in which the field theory can study these states: (1) in close proximity of the self-duality with preservation of the balance between the gauge field-induced repulsion and of the two-body attraction carried by the matter self-interaction term (for vortices of the same sign); we have derived equations that generalize the Abelian model; (2) without preservation of the balance mentioned above; in these cases it appears possible to simulate in a global way the dissipative events of reconnection of the streamlines, i.e. the processes that lead to higher topological order. It results that at every (dissipation-mediated) reconnection event a small quantity of helicity is removed from the fluid. Since this is represented by the Chern-Simons term and at SD this becomes the square of the vorticity, the result appears as compatible with the known decay of the enstrophy. It also results that the enstrophy is the

Kullback Leibler entropy which appears to be suggestive for the direct spectral cascade of the enstrophy. Here we find a strange connection with the random matrices: the monopolar Euler vortex, extended to the whole plane (therefore not at self-duality) is given by the Painleve III equation, which, on the other hand, describes the statistical properties of an ensemble of Gaussian random matrices. Many other extensions and connections are possible.

In a special way we identify deep connections with the theory of Constant Mean Curvature surfaces in the ordinary 3D space. Every stationary flow corresponds to a surface, this is actually the result. Very restrictive theorems about the CMC surfaces have impact on the realizability in real world of some flows. Every Delaunay unduloid corresponds to a periodic chain of dipolar vortices of the fluid.

In a much wider analysis than the one limited to the statistical approach it results a strange connection with the fermionic theory of Nambu-Jona-Lasinio (NJL). Our field theory is bosonic and includes the Chern-Simons term, the carrier of vorticity arising from ExB-type motions and also includes the matter field self-interaction. But a fermionic theory coupled with a gauge field is also able to produce the term Chern-Simons. In addition, the vacuum state of the FT version of the fluid has chiral charge +/-2. The ideal fluid is an example in the real world of spontaneous breaking of the chiral symmetry. The NJL system is massless but the fermions get a mass through the nonlinear effect, no need of the Higgs mechanism. This is an interesting field of parallel (fluid – NJL field theory) investigations.

We have naturally extended the field theory to two-dimensional fluids whose dynamics takes place on a background of global rotation, externally imposed. It actually is the class of systems with the widest application to real world: plasma in strong magnetic field, the planetary atmosphere, the oceans, the non-neutral plasma, etc. Here the equations of the discrete system of point-like vortices exhibit short-range interaction. The interaction between elements of the fluid's vorticity is altered by the presence of the background rotation. The field theory that we have constructed for the Euler fluid has had to be adapted to reflect the short range, via the Higgs mechanism: the photon of the gauge field gets a mass which is due to the condensate of vorticity imposed by the global rotation. We have explored two directions.

In the first direction of works, the dynamical vorticity is of single sign and the basic structure is monopolar. We have derived the equations of the stationary states. The solutions have the shape of a ring of vorticity, which suggests applications to „ocean rings". These are ocean flow formations, extremely stable and have ring-type geometry (they appear for example in Caraibe and drift for years). The field theory finds indeed the particular stability and connects it with the topological properties. The particularity of these solutions is that they have –ideally – a singularity of the azimuthal velocity in the center of the ring. The practical consequence is very interesting: a transversal motion becomes inevitable and we can imagine a wide range of applications (e.g. axial jets in the planar galaxies).

We have tried, in an independent direction, to extend the field theory to the systems that have vorticities of both signs (on the background of global rotation). Here we cannot find self-duality and we had to abandon the extremum of the action functional: there is a residual energy and this may suggest non-stationarity. We cannot say from the derivation how far from reality will be the solutions of the equation that we have derived: we just expect it to be indicative. Strangely, this equation has proved to be able to describe quantitatively well the stationary vortices of the 2D planetary atmosphere. We have derived two equations that connect basic characteristics of the tropical cyclones: the maximum azimuthal velocity, the „eye-wall" radius and the radius of the maximum extension of the vortex. We proved that we can provide results that have good comparison with the observation for tropical cyclone (NW Pacific, since they can reach stationarity) and also the radial profile of the azimuthal wind velocity. A result of the field theory: the equality of the mass of the matter-field excitation (particle) with the mass of the photon is translated, in the case of the tropical cyclones, into a statement regarding the geometry of flow: there is equality between the maximal radial extension and the Rossby radius.

The field theory reveals another strange connection, possibly source of future theoretical applications: in plasma placed in strong magnetic field the ion polarization drift allows, in some condition, the concentration of the vorticity. We see here a parallel with the axial anomaly, which means that the axial current (along a filament of vorticity in plasma) has non-zero divergence, measured by the topological degree of the integrated history (space and time) of the gauge field. In plasma we have the familiar picture of an injection of helicity (that in FT can be attributed to the Chern Simons term with time dependence) and this is physically equivalent with the well-known equation connecting the time variation of the vorticity with the parallel derivative of the parallel current (parallel here means along the strong magnetic field in which the plasma is embedded).

Besides the study of generation of asymptotic ordered states, the Project has proposed an original approach to the problem of generation of quasi-coherent structures in turbulent states of fluids and plasmas. These structures have spatial scale from small to intermediate (mesoscopic) and they initiate the complex evolution that lead to coherent flows of large spatial scales that we find in asymptotic regimes. These small structures have been observed in numerical simulations but the analytical description has had to be limited to phenomenological description.

These studies are based on the original approach results obtained in the Plasma Theory Group on the statistics of test particle statistics in stochastic velocity fields. We have developed a semi-analytic method that allows the study of the intrinsic trapping that exists in 2D incompressible turbulence. We have shown that the trapping of vertical type produces effects of memory, anomalous transport regimes and non-Gaussian distribution of the displacements. The trapped trajectories form quasi-coherent vortical structures. This method has been proven useful and efficient leading to a series of clarifications in problems of particle and energy transport in magnetically confined plasmas.

The project has proposed: (a) to extend the studies of the test particle type to other domains, (b) to evaluate the effect of the trajectory trapping on the evolution of the turbulence. The Objective (b) is very ambitious, since it involves the development of theoretical methods that should work for strongly nonlinear regimes, which implies to go beyond the existing theoretical methods that apply only to weak turbulence.

For the Objective (a), we have studied two problems of current interest: the stochastic advection of the vapors in the atmospheric convection leading to cumulus clouds [6, 12, and 13] and the diffusion of charged particles in the stochastic magnetic field of space plasmas [7, 9, 16, and 23].

Regarding the Objective (b) we have finalized researches that yielded an important result [4, 24], which consists of proving that trapping has a decisive effect in the nonlinear evolution of turbulence. Specifically, we have studied drift wave turbulence in confined plasma. We have shown that the nonlinear processes detected in numerical simulations (the increase of the correlation length of the potential – the inverse cascade, the generation of zonal flows and the nonlinear damping of the turbulence) are all connected to the ion trapping. This is the first analytical treatment based on first principles that leads to results compatible with the numerical simulation in the far nonlinear regimes. First results have also been obtained for ideal fluid turbulence described by pint-like vortices in interaction [19].

We finally mention that three students with eminent results have been hired and are now permanent employees.

We have submitted eight Project Proposals to European competitions, of which five as Principal Investigator. Two projects have been accepted (one with European partial support). Four Projects are still under evaluation.

The results have been communicated in six articles ISI (four published, one accepted and one submitted); four electronic preprints have published on http://arXiv.org , and others are in preparation. There were 14 communications to Conferences, of which four have been invited talks.